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On the Complexity of Drawing Trees Nicely: Corrigendum

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Abstract. In this journal, Supowit and Reingold [1] have given a proof that it is NP-complete to decide whether a binary tree can be drawn on a grid with width 24 if certain æsthetic requirements are obeyed. We identify and repair a mistake in their proof.

1 Problem

We have identified a mistake in the proof in [1] that it is NP-complete to decide whether a binary tree can be drawn on a grid with width 24 if certain æsthetic requirements are obeyed. This nice result implies that, given these æsthetic requirements, it is NP-hard to approximate the minimum width of binary tree drawings up to about 4%. The purpose of this short note is to present a correct proof of this result that is, in the meantime, "folklore" in the automatic graph drawing community. We do not give the entire proof but rather refer to the arguments and the notation used in [1] while repairing the proof.

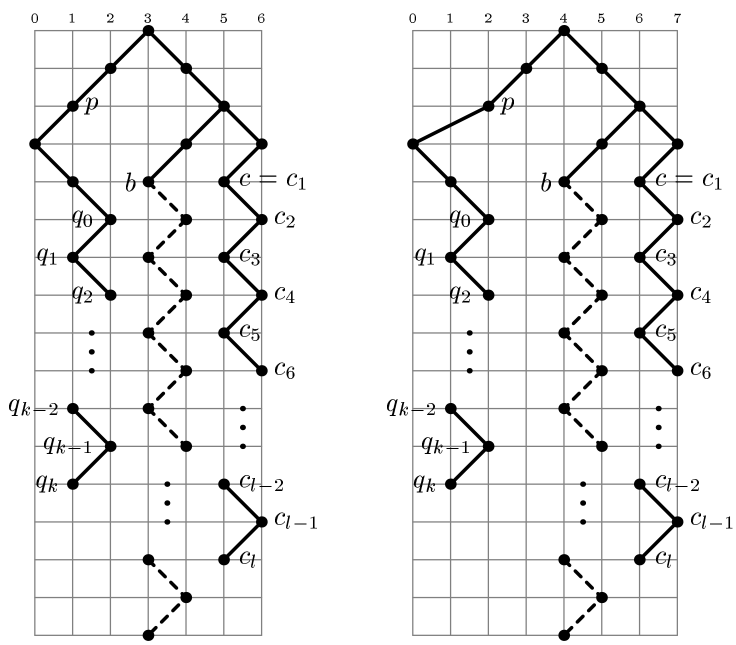
The NP-completeness proof in [1] is based on a transformation of a 3-SAT formula $E = F_1 \wedge F_2 \wedge \cdots \wedge F_r$ into a binary tree $T(E)$ such that $T(E)$ can be drawn on a grid with width 24 if and only if E is satisfiable. The key to success is that a subtree corresponding to a literal that evaluates to *false* needs width 7 whereas a subtree corresponding to a literal that evaluates to *true* needs only width 6.

One of the æsthetic requirements says that isomorphic subtrees must be drawn identically up to translation. Unfortunately, the given construction violates this requirement as it builds on drawings in which the subtrees rooted in column 4 at depth 2 in Fig. 9 (a/b) of [1] are drawn differently. A further source of incorrectly drawn isomorphic subtrees is the following: If the same literal occurs in different clauses, the corresponding subtrees, extended by one or two edges above their roots, may have to be drawn differently due to the construction of the clause trees $CT(F)$ as defined in [1] and exemplified in Fig. 10 of [1].

2 Solution

The proof can be repaired by different definitions of the literal trees as given in Figs. 1 and 2 that replace Figs. 8 and 9 in [1], respectively. They differ from the original versions as follows:

- The nodes labelled "b" take the rôle of the nodes labelled "w" in [1].
- The nodes labelled "c" are the roots of "zigzagging tails" of lengths $l \in \{1, 2, \dots, 3r\}$. A literal y_{ij} ($i \in \{1, 2, \dots, r\}$, $j \in \{1, 2, 3\}$) receives the unique identification $l = 3(i-1) + j$.



(a) (b)

Fig. 1 The new version of the literal tree $LT(y)$ where $y = x_k$

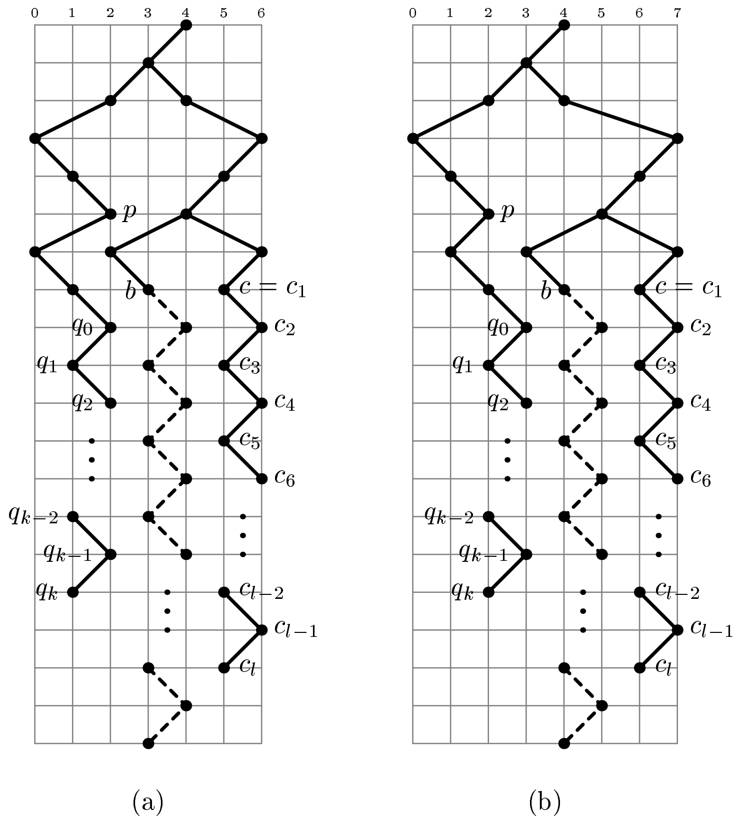


Fig. 2 The new version of the literal tree $LT(y)$ where $y = \overline{x}_k$

The "zigzagging tail" that is rooted at node b if the literal is a "middle literal" in its clause (drawn dashed in Figs. 1 and 2) extends two levels beyond the longest subtree rooted at "p"- or "c"-nodes in a given clause except the last clause F_r. Therefore, the length (in terms of the number of nodes) of any "zigzagging tail" rooted at a "b"-node is bounded by $\max\{n+4, 3r+2\}$ and this makes sure that the construction of $T(E)$ remains a polynomial time construction in the input size.

The new version corrects (and simplifies) the construction given in [1]. It makes sure that the "draw isomorphic subtrees identically up to translation" æsthetic essentially applies only to the variable subtrees of the entire tree constructed in the transformation as has been clearly intended by the authors of [1].

The example drawing of a clause tree $CT(F)$ displayed in Fig. 10 of [1] must then be modified to the version shown in Fig. 3, assuming that $F = F_1$, i.e., F is the first clause in the 3-SAT formula E.

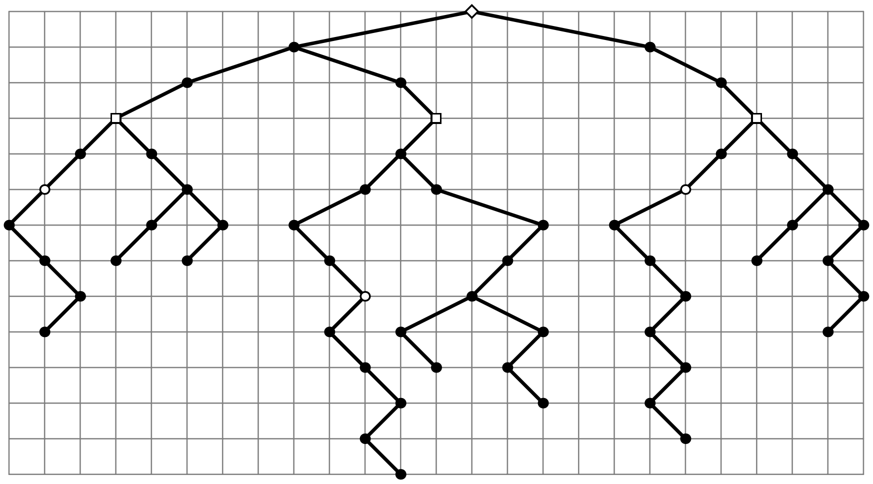


Fig. 3 The new version of the clause tree $CT(F_1)$ where $F_1 = (x_1 + \overline{x}_2 + x_4)$

The example displayed in Fig. 11 of [1] is modified as shown in Fig. 4.

References

1. Kenneth J. Supowit and Edward M. Reingold, The Complexity of Drawing Trees Nicely, Acta Informatica **18** (1983) 377–392.

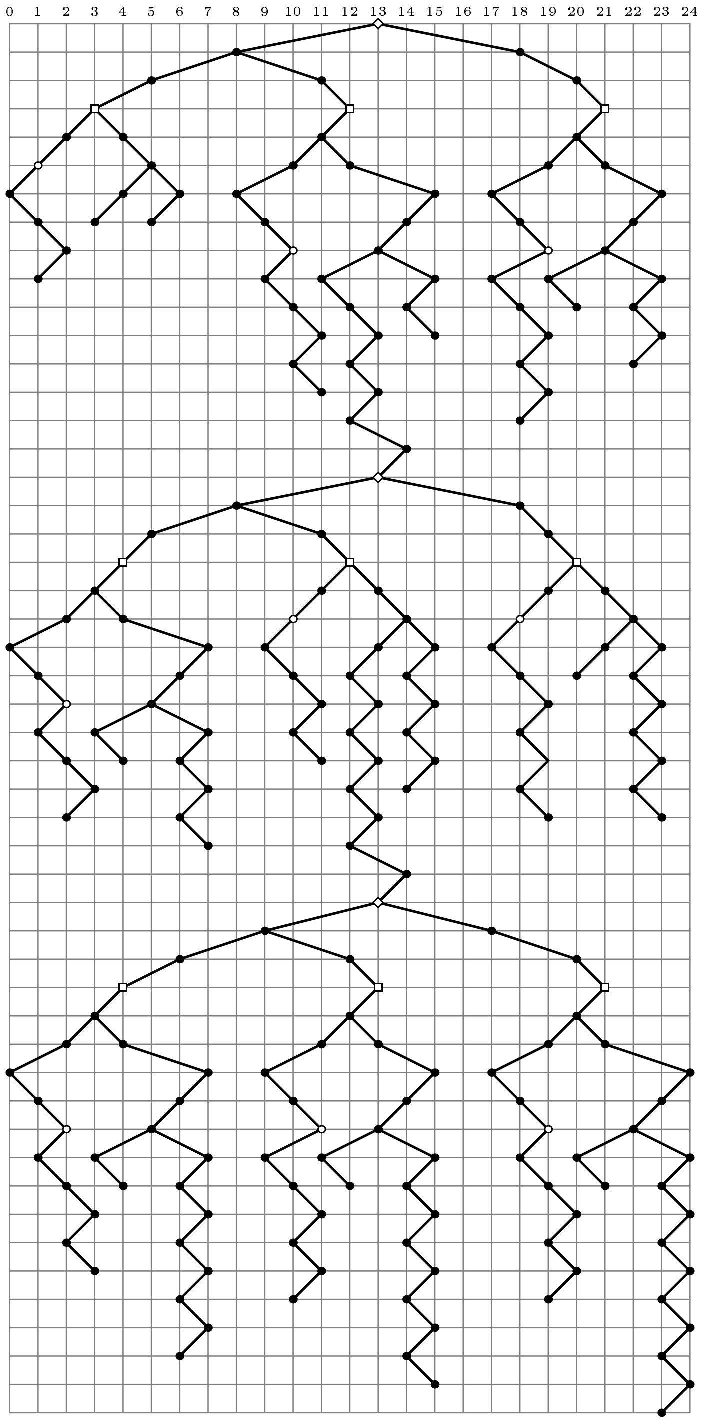


Fig. 4 The new version of the example given in Fig. 11 of [1]